Capture

In[1]:=
```
<< Graphics`Graphics`;
th = 0.007;
GStandardOptions := SetOptions[{Plot, ListPlot, LogPlot, LogLogPlot, ParametricPlot},
   Frame → True, AspectRatio → 1 / GoldenRatio, PlotRange → All,
   PlotRegion → Automatic, Axes → True, AxesStyle → Automatic,
   PlotStyle → {{RGBColor[0.5, 0.5, .5], Thickness[th]}, {RGBColor[1, 0, 0], Thickness[th]},
     {RGBColor[0, 1, 0], Thickness[th]}, {RGBColor[0, 0, 1], Thickness[th]}},
   TextStyle → {FontSize → 18, FontWeight → "Bold"}];
GStandardOptions;
DispOn := SetOptions[Plot, DisplayFunction → $DisplayFunction];
DispOff := SetOptions[Plot, DisplayFunction → Identity];
<< Graphics`Colors`
<< Miscellaneous`PhysicalConstants`
<< Graphics`Legend`
```

Basic Formalism

The process is described in two steps

(1) μ + (A,Z) → (A,Z-1)* + ν

then

(2a) (A,Z-1)* → (A,Z-1) + γ (10%).

(2b) A,Z-1)* → (A-1,Z-1) + n ... (90 %)

i.e. roughly 10% of nuclei remain bound and 90% have 1 n (or 2 n or charged particle emission).

Let us consider such a simple two stage model. The cross section is dominated by the isobaric analogs to the photonuclear giant dipole resonance, with a nuclear excitation energy of ~22 MeV. Accordingly the neutrino momentum q in reaction (1) is ~85 MeV/c (with a width reflecting the giant resonance width).

In[16]:=
```
q = 85; m0 = 931.5; mn = 939.6;
en = 1;
vecN = {"N", 14 m0, q^2/(2 14 m0), q/(14 m0)};
vecO = {"O", 16 m0, q^2/(2 16 m0), q/(16 m0)};
vecn = {"n", mn, en, Sqrt[2 en/mn]};
MatrixForm[{vecN, vecO, vecn}]
```

Out[21]//MatrixForm=

$$\begin{pmatrix} \mathrm{N} & 13041. & 0.277011 & 0.00651791 \\ \mathrm{O} & 14904. & 0.242385 & 0.00570317 \\ \mathrm{n} & 939.6 & 1 & 0.0461364 \end{pmatrix}$$

According to Peter W. EH~0.085 MeV and EVH~0.35 MeV. So the EVH should cut significantly into this naive energy spectra and apparently it does not, for N.

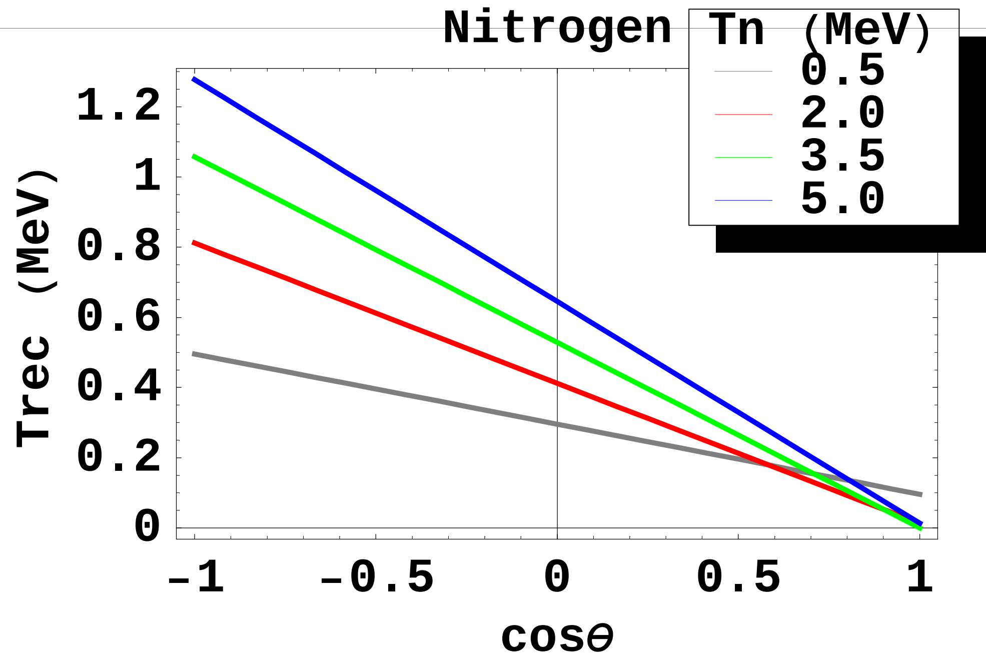
We denote lab variables with small and nucleus CMS variables with capital letters. The next step is to calculated the spectral shape because of the emission of a neutron of energy tn. Here we assume the nucleus is at maximum velocity vi = vm = $\frac{q}{M}$ and vi=0. As the neutron velocity vn is large against the velocity of the nucleus vi, we set tn=Tn or vn=Vn. If the n is emitted with u=cosθ in the nucleus CMS moving with velocity vi, the nuclear recoil energy will be

In[22]:=
```
t0[M_, vi_, Tn_, u_] :=
 (vn = Sqrt[2 Tn/mn]; vrec = vn mn/(M - mn); vf = Sqrt[vi^2 + vrec^2 - 2 vi vrec u]; (M - mn) vf^2/2)
```

In[87]:=
```
M = vecN[[2]]; vi = vecN[[4]]; {M, vi};
Tn = {0.5, 2., 3.5, 5.};
MatrixForm[Table[{t0[M, vi, Tn[[i]], 1], vi, vrec}, {i, 1, 4}]]
Plot[{t0[M, vi, Tn[[1]], u], t0[M, vi, Tn[[2]], u], t0[M, vi, Tn[[3]], u],
   t0[M, vi, Tn[[4]], u]}, {u, -1, 1}, FrameLabel → {"cosθ", "Trec (MeV)"},
  PlotLegend → {"0.5", "2.0", "3.5", "5.0"},
  LegendLabel → "Tn (MeV)",
  LegendPosition → {.4, .2},
  LegendSize → {.5, .4},
  PlotLabel → "Nitrogen"
 ];
M = vecO[[2]]; vi = vecO[[4]]; {M, vi};
MatrixForm[Table[{t0[M, vi, Tn[[i]], 1], vi, vrec}, {i, 1, 4}]]
Plot[{t0[M, vi, Tn[[1]], u], t0[M, vi, Tn[[2]], u], t0[M, vi, Tn[[3]], u],
   t0[M, vi, Tn[[4]], u]}, {u, -1, 1}, FrameLabel → {"cosθ", "Trec (MeV)"},
  PlotLegend → {"0.5", "2.0", "3.5", "5.0"},
  LegendLabel → "Tn (MeV)",
  LegendPosition → {.4, .2},
  LegendSize → {.5, .4},
  PlotLabel -> "Oxygen"
 ];
```

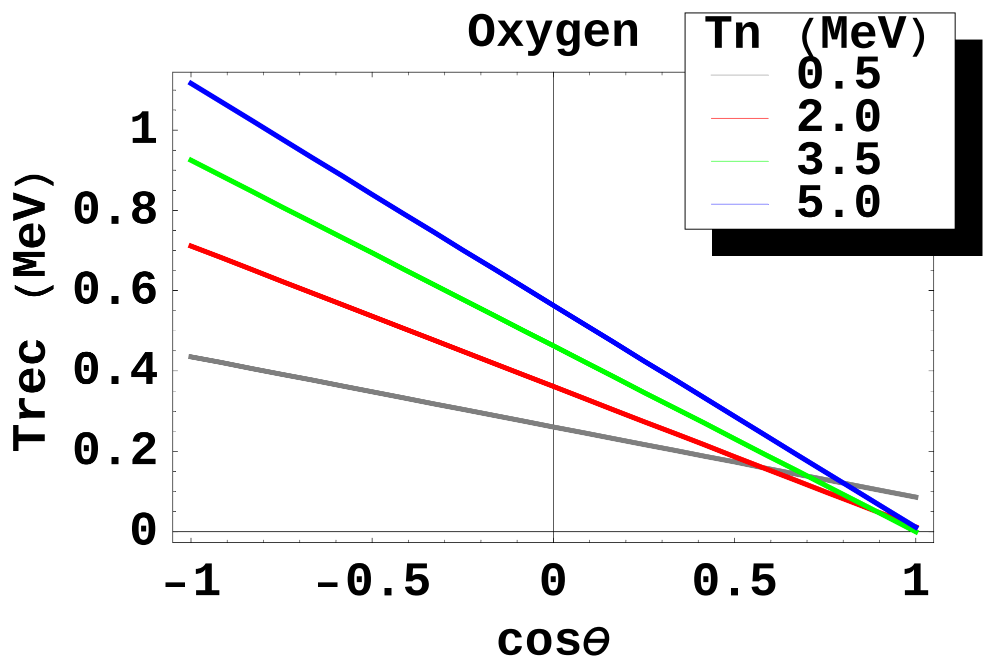
Out[89]//MatrixForm=

$$\begin{pmatrix} 0.0960817 & 0.00651791 & 0.002533 \\ 0.0127549 & 0.00651791 & 0.00506601 \\ 0.000204394 & 0.00651791 & 0.0067017 \\ 0.0134721 & 0.00651791 & 0.00801006 \end{pmatrix}$$

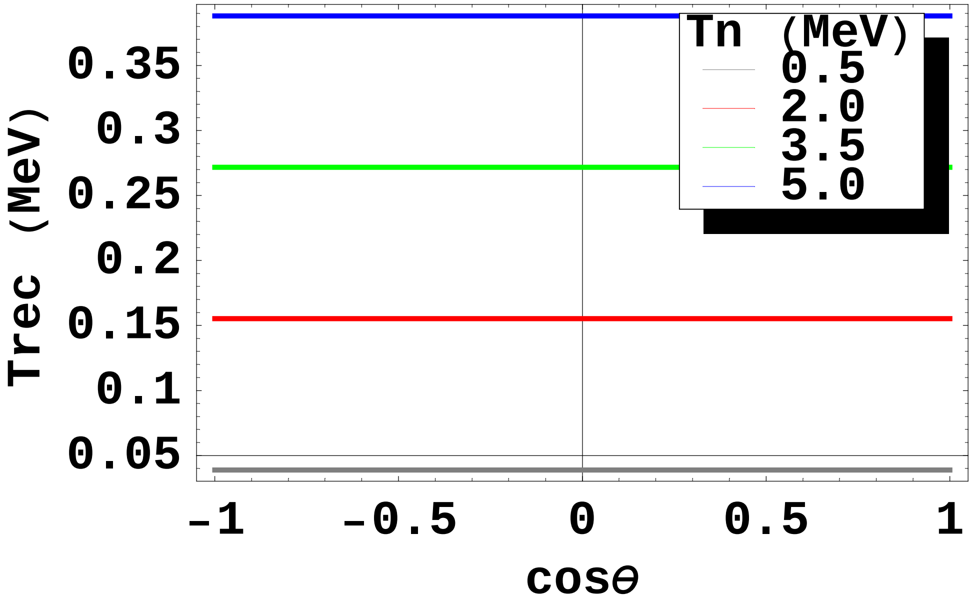


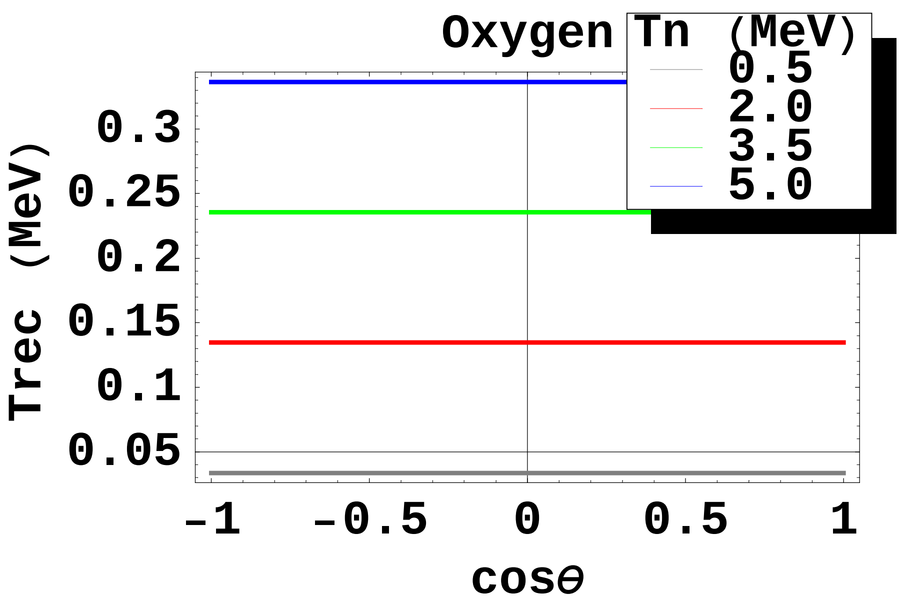
Out[92]//MatrixForm=

$$\begin{pmatrix} 0.0859279 & 0.00570317 & 0.00219507 \\ 0.0120374 & 0.00570317 & 0.00439015 \\ 0.0000761799 & 0.00570317 & 0.00580762 \\ 0.0107059 & 0.00570317 & 0.00694143 \end{pmatrix}$$



```
In[94]:= M = vecN[[2]]; vi = 0; {M, vi};
Plot[{t0[M, vi, Tn[[1]], u], t0[M, vi, Tn[[2]], u], t0[M, vi, Tn[[3]], u],
   t0[M, vi, Tn[[4]], u]}, {u, -1, 1}, FrameLabel → {"cosθ", "Trec (MeV)"},
  PlotLegend → {"0.5", "2.0", "3.5", "5.0"}, LegendLabel → "Tn (MeV)",
  LegendPosition → {.4, .2}, LegendSize → {.5, .4}, Label → "Nitrogen"];
M = vecO[[2]]; vi = 0; {M, vi};
Plot[{t0[M, vi, Tn[[1]], u], t0[M, vi, Tn[[2]], u], t0[M, vi, Tn[[3]], u],
   t0[M, vi, Tn[[4]], u]}, {u, -1, 1}, FrameLabel → {"cosθ", "Trec (MeV)"},
  PlotLegend → {"0.5", "2.0", "3.5", "5.0"}, LegendLabel → "Tn (MeV)",
  LegendPosition → {.4, .2}, LegendSize → {.5, .4}, PlotLabel → "Oxygen"];
```





In conclusion this study just emphasizes the old issue of nuclear physics uncertainties in making predictions on the relative N and O recoil spectra.

i) From the simple 2-body kinematics both N and O should be cut by the significantly cut by the EVH threshold. However, even this spectrum is uncertain, as it depends on the detailed energy dependence of the capture rate in the giant resonance region.

ii) The additional recoil generated by neutron evaporation of the excited nucleus might significantly modify the recoil energy

Thus, before we have analyzed empirical recoil spectra, we cannot make assumptions on the relative N vs O recoil spectra. In particular, the assumption ϵH(oxygen) $\approx$ ϵN(nitrogen) cannot be justified.

References

http://www.npl.uiuc.edu/twiki/bin/view/Main/CaptureFADC has a literature reference to neutron emission spectra after capture in C and O.

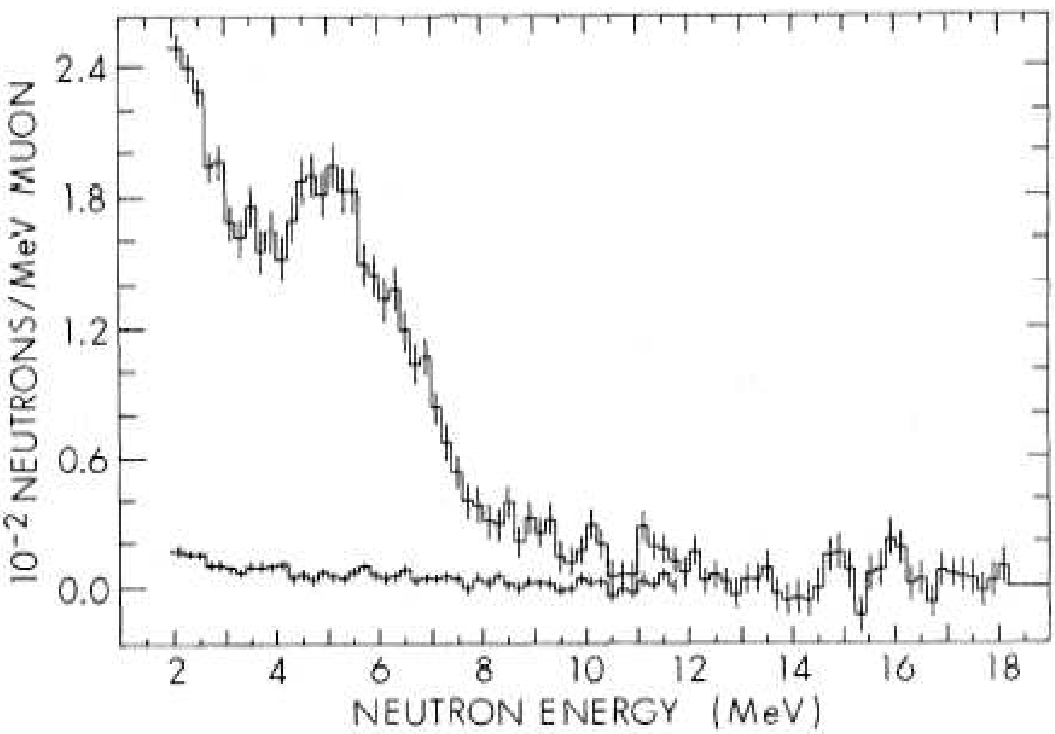


FIG. 2. Unfolded neutron spectrum and background for ^{16}O.

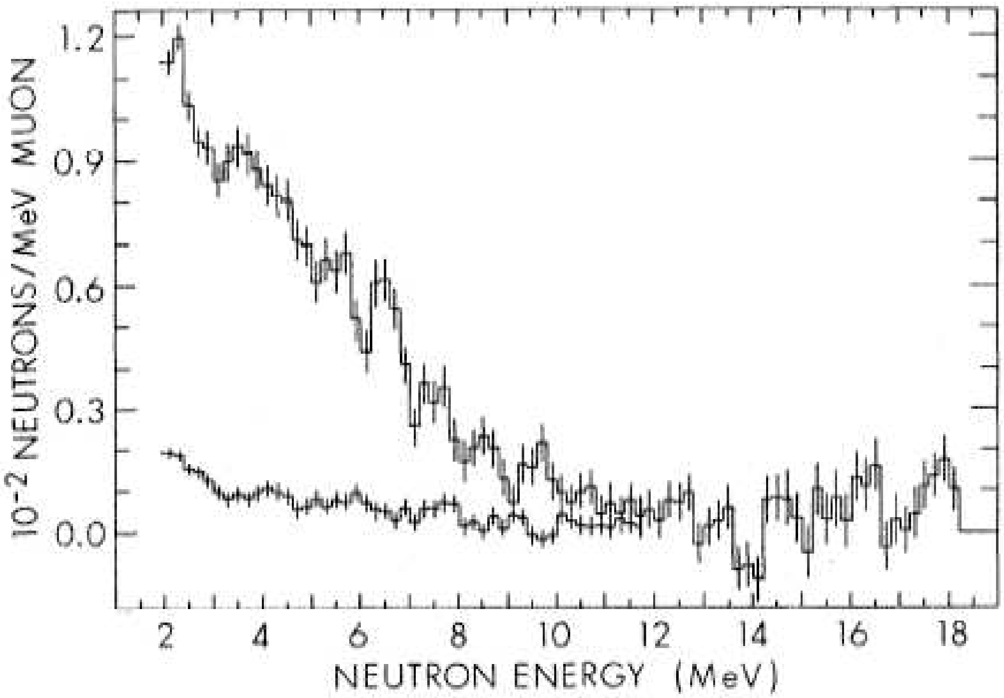


FIG. 6. Unfolded neutron spectrum and background for ^{12}C.