

# RMC a'la Kauland Thesis

<http://mu2e-docdb.fnal.gov:8080/cgi-bin/ShowDocument?docid=521>

## Initialize

### 3.3.3 The radiative muon capture process

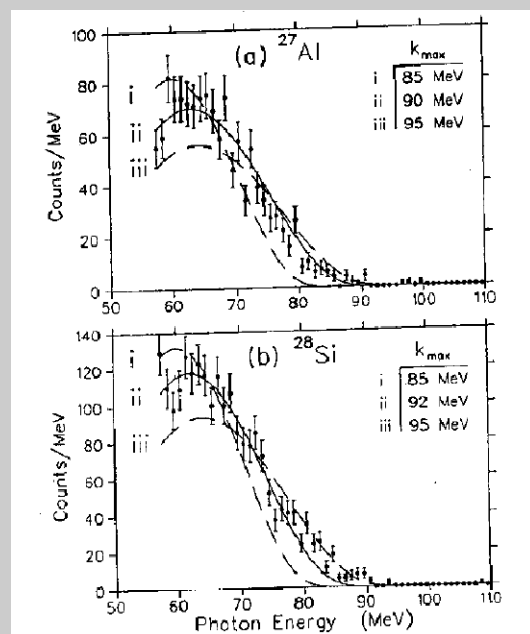
$$k^{\max, \text{calc}} = m_{\mu} - \Delta M - m_e - R_{Z-1, A} - B_{\mu}$$

with  $\Delta M$  nuclear mass difference,  $R_{Z-1, A}$  recoil energy,  $B_{\mu}$  1s binding energy.

Primakoff formula

```
Clear[k, kmax];
```

```
f[k_, kmax_] := (x = k / kmax; If[x < 1, 20 (1 - 2 x + 2 x^2) (1 - x)^2, 0.] )
```

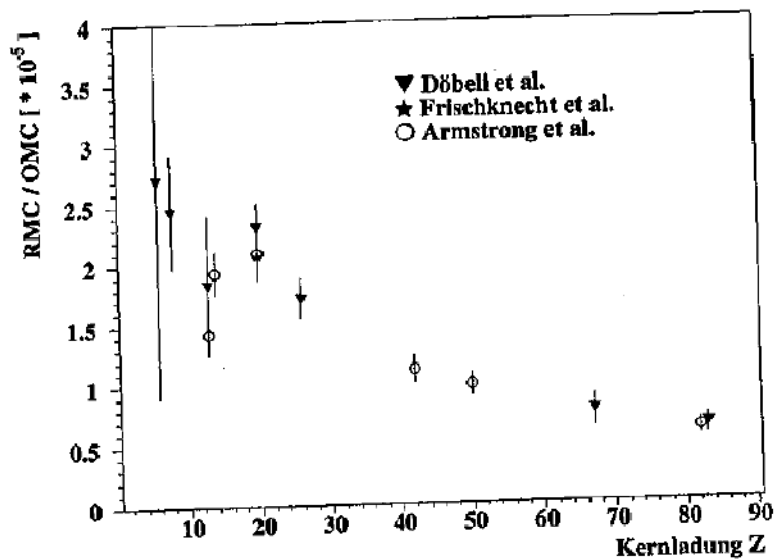


A	rel Anteil in [%]	$m_{Ti}$ in [MeV/c <sup>2</sup> ]	$m_{Sc}$ in [MeV/c <sup>2</sup> ]	$E_{\mu}^{\max, \text{calc}}$ in [MeV]	$E_{\mu}^{\max, \text{Ca}}$ in [MeV]	$E_{\mu}^{\max, \text{e}^+}$ in [MeV]
46	7.94	42804.95	42807.3	100.88	91.38 ± 2	102.26
47	7.44	43735.64	43736.2	102.65	93.15 ± 2	100.66
48	73.78	44663.56	44667.6	96.24	89.74 ± 2	98.99
49	5.51	45595.02	45597.0	101.25	91.75 ± 2	95.98
50	5.34	46523.65	46530.5	96.35	86.88 ± 2	91.39

Massen nach [10]  
relative Anteile nach [51]

The Primakoff formula describes the shape of the measured RMC spectra, if  $k_{\max}$  is taken as a fit parameter. For Ca one finds a  $k_{\max, \text{Ca}}$  which is 9.5 MeV lower than  $k_{\max}$  calculated. It is assumed that shift corresponds to some average nuclear excitation. In the table above the same shift is also assumed for the other elements. The Primakoff Model overestimates the RMC rate.

The experimental fraction of RMC/OMC for  $k > 57$  MeV is estimated to be  $2.0 \pm 0.5 \cdot 10^{-5}$ .

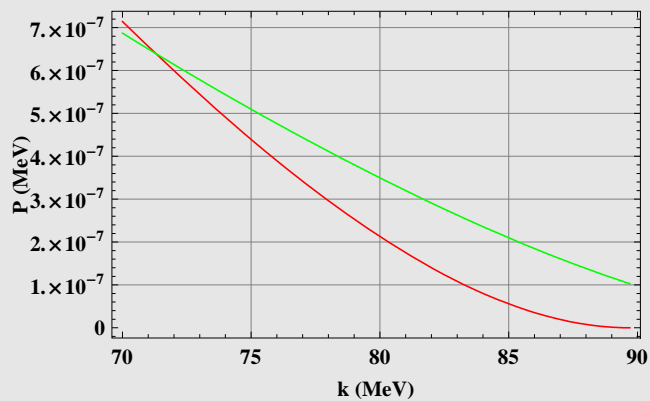


#### 5.4 Simulation of RMC

```

kmc = 99.2; kmca = 89.7 ; (* A=48 *)
fscal = 2 10-5;
fsc = fscal / Integrate[f[k, kmc], {k, 57, kmc}];
fsca = fscal / Integrate[f[k, kmca], {k, 57, kmca}];
Plot[{f[k, kmca] fsca, f[k, kmc] fsc},
  {k, 70, kmca}, FrameLabel -> {"k (MeV)", "P (MeV)"}]
ScientificForm[{Integrate[f[k, kmca] fsca, {k, 80, kmca}] ,
  Integrate[f[k, kmc] fsc, {k, 57, kmc}]}]

```

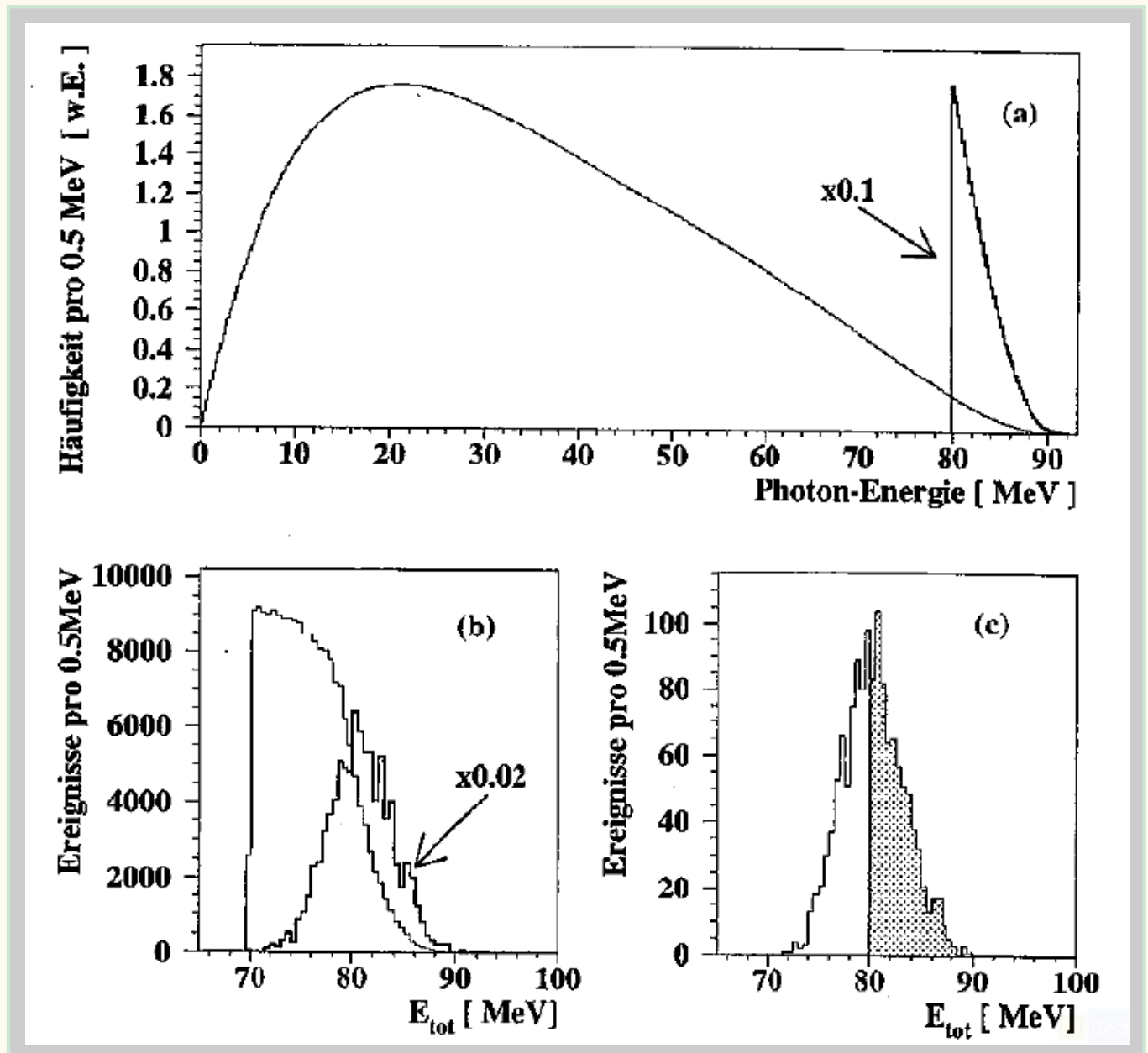


```
{7.25836 x 10-7, 2. x 10-5}
```

Kauland gives  $P_{\gamma}(k > 80 \text{ MeV}) = 1.02 \cdot 10^{-6}$  per OMC. I get  $0.7 \cdot 10^{-6}$ . For positrons this MC gives  $P_e(k > 80 \text{ MeV}) = 2.6 \cdot 10^{-11}$  per OMC, for the SINDRUM setup.

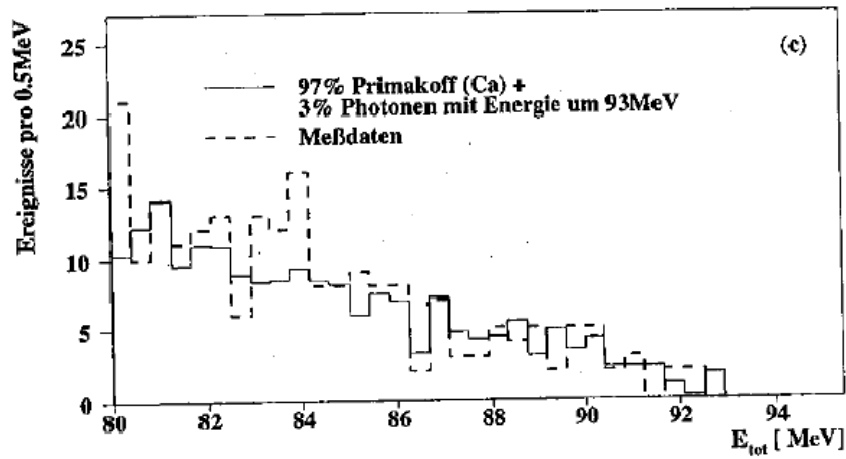
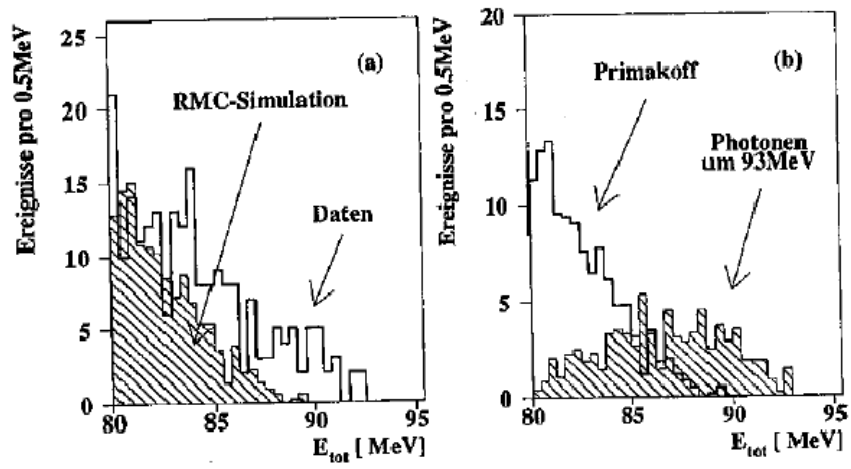
```
PosPerRMC = 2.6 10-11 / 10-6; ScientificForm[PosPerRMC]
```

```
2.6 x 10-5
```



### 7.3 Discussion RMC continuum

Kauland finds more high energy positrons than explained by the Primakoff formula. If one assumes a 3% contribution of  $\sim 93$  MeV gammas, the positron spectrum can be fitted.



Radiative muon capture on O, Al, Si, Ti, Zr, and Ag  
 Bergbusch, P.C. et al., Physical Review C 59, 2853-64, 1999

#### D. RMC in Ti as a background to $\mu$ - $e$ conversion

RMC is a major background for  $\mu$ - $e$  conversion experiments, many of which use Ti as a target [39]. This background arises from the asymmetric conversion of RMC photons in which a high energy positron is produced. Before now, RMC on Ti had never been measured so the experimental branching ratio for Ca was used, corrected by a factor of 0.91 to account for the  $Z$  dependence of the RMC to OMC ratio. To fit the PSI data [39], a Primakoff polynomial spectral shape was assumed and applied to each Ti isotope. The end point photon energy for each isotope was determined by subtracting from the kinematical RMC end point the same amount subtracted in the Ca case. This was found to be insufficient to describe the observed background; so an additional spectrum with a 93 MeV end point was added.

The present measurement of the RMC rate in Ti ( $R_\gamma = 1.30 \times 10^{-5}$ ) is significantly lower than the  $1.93 \times 10^{-5}$  assumed in Ref. [39]. Also, the best fit of the end point energy  $k_{\max}$  to the spectrum is 89.2 MeV, a bit lower than the as-

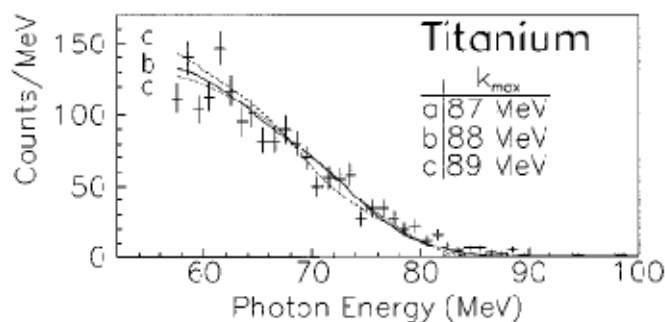


FIG. 14. Comparison of the experimental RMC on Ti photon spectrum (error bars) with the closure spectral shape of Eq. (7) after convolution with the spectrometer response and software cuts. The solid line is the spectral shape for the best fit value of  $k_{\max}$ .

sumed value of 89.7 MeV for the dominant Ti isotope. As shown in Fig. 14, there appears to be no evidence of RMC photons between 89 and 93 MeV, which implies that the observed background in  $\mu$ - $e$  conversion does not result from the RMC process.

TABLE XIV. Summary of the world data on the quantity  $R_\gamma$  on  $A > 3$  targets, where  $R_\gamma$  is the ratio of the radiative muon capture rate with  $E_\gamma > 57$  MeV to the ordinary muon capture rate in units of  $10^{-5}$ . In order to assist comparisons in most cases the quoted results are for the closure approximation spectra shape where the corresponding value of the parameter  $k_{\max}$  is given in column 4. The quantity  $\alpha$  is the neutron excess  $(A - 2Z)/Z$ .

Target	$\alpha$	$R_\gamma (\times 10^{-5})$	$k_{\max}$ (MeV)
<i>Bergbusch et al. (1999)</i>			
$^{16}_8\text{O}$	0.000	$1.67 \pm 0.18$	$88.4 \pm 2.3$
$^{27}_{13}\text{Al}$	0.077	$1.43 \pm 0.11$	$90.1 \pm 1.8$
$^{28}_{14}\text{Si}$	0.000	$2.09 \pm 0.20$	$89.4 \pm 1.8$
$^{nat}_{22}\text{Ti}$	0.173	$1.30 \pm 0.12$	$89.2 \pm 2.0$
$^{nat}_{40}\text{Zr}$	0.280	$1.31 \pm 0.15$	$89.2 \pm 3.4$
$^{nat}_{47}\text{Ag}$	0.296	$1.12 \pm 0.13$	$89.0 \pm 3.2$
<i>Gorringe et al. (1998)</i>			
$^{58}_{28}\text{Ni}$	0.071	$1.48 \pm 0.08$	$92.0 \pm 2.0$
$^{60}_{28}\text{Ni}$	0.143	$1.39 \pm 0.09$	$90.0 \pm 2.0$
$^{62}_{28}\text{Ni}$	0.214	$1.05 \pm 0.06$	$89.0 \pm 2.0$
<i>Armstrong et al. (1992)</i>			
$^{27}_{13}\text{Al}$	0.077	$1.43 \pm 0.13$	$90.0 \pm 2.0$
$^{28}_{14}\text{Si}$	0.000	$1.93 \pm 0.18$	$92.0 \pm 2.0$
$^{40}_{20}\text{Ca}$	0.000	$2.09 \pm 0.19$	$93.0 \pm 2.0$
$^{nat}_{42}\text{Mo}$	0.283	$1.11 \pm 0.11$	$90.0 \pm 2.0$
$^{nat}_{50}\text{Sn}$	0.374	$0.98 \pm 0.09$	$87.0 \pm 2.0$
$^{nat}_{82}\text{Pb}$	0.527	$0.60 \pm 0.07$	$84.0 \pm 3.0$
<i>Armstrong et al. (1991)</i>			
$^{12}_6\text{C}$	0.000	$1.98 \pm 0.20$	
$^{16}_8\text{O}$	0.000	$2.18 \pm 0.20$	
$^{40}_{20}\text{Ca}$	0.000	$2.04 \pm 0.14$	
<i>Frischknecht et al. (1988)</i>			
$^{16}_8\text{O}$	0.000	$3.80 \pm 0.40$	
<i>Döbeli et al. (1986)</i>			
$^{12}_6\text{C}$	0.000	$2.70 \pm 1.80$	
$^{16}_8\text{O}$	0.000	$2.44 \pm 0.47$	$89.9 \pm 5.0$
$^{27}_{13}\text{Al}$	0.077	$1.83 \pm 0.26$	$88.8 \pm 1.8$
$^{40}_{20}\text{Ca}$	0.000	$2.30 \pm 0.21$	$92.5 \pm 0.7$
$^{nat}_{26}\text{Fe}$	0.146	$1.71 \pm 0.17$	$90.2 \pm 1.1$
$^{165}_{67}\text{Ho}$	0.463	$0.75 \pm 0.13$	$84.1 \pm 5.1$
$^{209}_{83}\text{Bi}$	0.518	$0.62 \pm 0.08$	$88.2 \pm 0.6$
<i>Frischknecht et al. (1985)</i>			
$^{40}_{20}\text{Ca}$	0.000	$1.92 \pm 0.20$	$90.8 \pm 0.9$
<i>Hart et al. (1977)</i>			
$^{40}_{20}\text{Ca}$	0.000	$2.11 \pm 0.14$	$86.5 \pm 1.9$