Summary:

1. Steve's $f = \frac{\varepsilon_{but}(0) - \varepsilon_{but}(R_d)}{\varepsilon_{but}(0)}$ functions in table I seem to factorize

$$f(b_{cut}, R_d) = g(b_{cut}) \cdot h(R_d) \qquad (1)$$

(the approximations to this relation need to be quantified)

2. Under approx. (1) Steve's equ. 28 can be reformulated

$$I_t = S_0 + \int_0^\infty dt\, t\, \lambda_0 e^{-\lambda_0 t} \int_0^\infty dR_d\, F_{mp}(R_d)\, f(b_{cut}, R_d)$$

$$= S_0 + g(b_{cut}) \cdot \int \quad \int dR_d\, F_{mp}(R_d)\, h(R_d)$$

$$= S_0 + g(b_{cut}) \langle t \rangle$$

The last integral now is independ to b_{cut}, universal.

With normalization of $I_{Norm} = 1 + g(b_{cut}) \langle \; \rangle$, so that $\langle \; \rangle = 1$, we get

$$\Delta S(b_{cut}) = g(b_{cut}) \left(S_0 - \langle t \rangle\right) \qquad (2)$$

It thus follows that, independent of the diffusion model, the b_{cut} dependence of ΔS is described by only one fit parameter

$$\left(S_0 - \langle t \rangle\right) \qquad (3)$$

All models having identical values for (3) are indistinguishable.

3. Can we understand (1) from Steve's impact parameter derivation Eq. 19 ..

$$w(b) = \begin{cases} -1 + \Theta_b(b)/\pi & b \le b_{cut} \\ \Theta_b(b)/\pi & b > b_{cut} \end{cases} \qquad (4)$$

$$\cos\Theta_b = \frac{b^2 + R_d^2 - b_{cut}^2}{2 R_d b}$$ is reformulated in terms of $b \to b - b_{cut}$

We get $$\cos\Theta_b = \frac{1}{2R_d} \frac{b^2 - 2bb_{cut} + R_d^2}{b - b_{cut}}$$

$$= \frac{1}{R_d} \frac{b - \frac{b^2}{2b_{cut}} + \frac{R_d^2}{2b_{cut}}}{1 - \frac{b}{b_{cut}}} \approx \frac{1}{R_d}\left(b + \frac{b^2}{2b_{cut}} + \frac{R_d^2}{b_{cut}}\right.$$

$$\approx \frac{b}{R_d} + \frac{R_d}{2b_{cut}}\left(1 + \left(\frac{b}{R_d}\right)^2\right)$$

$b = R_d x$ $$\approx x + \frac{R_d}{2b_{cut}}(1 + x^2) \qquad (5)$$

$|x| < 1$, for $\frac{R_d}{2b_{cut}} \ll$ simple odd function

$$\Theta_b = \cos^{-1}\left(\frac{b}{R_d}\right) \qquad (6)$$

If we now expand $F_b(b) = F_b(b_{cut}) + b \cdot \frac{\partial F_b}{\partial b}(b_{cut}) + \frac{b^2}{2}\frac{\partial^2 F}{\partial b^2}$ we see that in the integral

$\Delta E_{cut}(R_d) = \int_{-R_d}^{+R_d} db\, w_b\, F_b(b)$ only odd functions in the expansion above contribute, leading to the simple approx.

(7) $\Delta E_{cut}(R_d) = \frac{\partial F_b}{\partial b}(b_{cut}) \int_{-R_d}^{R_d} db\, b\, w_b(b)$, as in equ. (1)

4. Check of Steve's integration.

The integration leading to equ. 4 and to the annular cuts is tricky. Thus I wrote a little MC simulation to check this. This effort is documented on the webpage.

5. How can we improve/check the precision of this ~~method~~ method?
I have no clear ideas, how to check the $\Delta\epsilon_{\text{bcut}}(R_d)$ part, apart from more MC work.

The $F_{\mu p}(R_d)$ function could be experimentally investigated.

$F_{\mu p}(R,t)$ can be measured as the distribution $T(R,t)$ of a transfer Auger to an impurity or as $C(R,t)$ of a capture on Xe (Ar?), if capture is nearly instantaneous.

Assume we introduce an impurity, such that a fraction 10^{-3}/muons transfer. With 10k ~~incident~~ stops and dominant capture this yields a rate $= 10k \cdot 10^{-3} \cdot \underset{0.2}{\epsilon} = 2$ Hz or 200k events/d

The spacial resolution will be ≈ 4 mm, 1 mm in y if we observe a capture n in coinc, but that's a significant loss in statistics.
Ideally we would reduce the TPC gas density to ~~en~~ widen the distribution by say 5x, @ 2 bar.
We need a MC/analytical study to clarify what can be learned from such a measurement.

6. μd determination.

Related to section 5 is the old idea of μd monitoring by Ar, N, Xe admixture

The sensitivity for N capture should be ~~around~~ better than 1 ppm yield.

If we mix in Z, so that a fraction of 10^{-2} transfers.

The μd transfer is $150\,s^{-1}\,C_d$, ~~30~~ $3 \cdot 10^{-4}\,C_d$ (ppm).

Thus total displaced captures are 3 ppm C_d (ppm), or 0.3 ppm @ 0.1 ppm.

Assume 10^{-7} can be observed. With

$10h \cdot 10^{-7} = \underline{10^{-3}}$ 3.6/hour

Pretty tough, not impossible.