Radiative Corrections to β Decay and the Possibility of a Fourth Generation

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Leading-logarithmic radiative corrections to β decay are summed via the renormalization group and structure-dependent $O(\alpha)$ effects are estimated by a form-factor analysis. These refinements reduce the Kobayashi-Maskawa quark-mixing-matrix element $|V_{ud}|$ by 0.13% to 0.9729 ± 0.0012. Combined with K_{e3}-, hyperon-, and b-decay constraints, this implies $|V_{ud}|^2 + |V_{us}|^2 + V_{ub}|^2 = 0.9954 \pm 0.0025$. Although consistent with unitarity at the 2σ level, our result leaves open the possibility of a fourth fermion generation which at 90% confidence level may have mixing $|V_{ub'}|$ as large as 0.088.

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The comparison of $0^+ \to 0^+$ superallowed Fermi β transitions and muon decay has played an important role in the development of the standard model of strong and electroweak interactions. The conserved-vector-current hypothesis, current algebra, Cabibbo universality, and a host of other theoretical advances have their roots in such studies. More recently, it has been shown[1] that experimental measurements of $\mathscr{F}t$ values in superallowed Fermi β decays, when combined with muon and K_{e3} (or hyperon) decay rates, provide a precision test of the standard $SU(2)_L \otimes U(1)$ model at the level of its quantum corrections. Indeed, neglecting radiative corrections, one finds[1]

$$|V_{ud}|^2 + |V_{us}|^2 \approx 1.032 \tag{1}$$

(without radiative corrections) for the first two elements of the Kobayashi-Maskawa[2] quark mixing matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \tag{2}$$

That would constitute a clear violation of unitarity which, for example, requires in the three-generation case

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1. \tag{3}$$

Fortunately, the $O(\alpha)$ radiative corrections which are finite and calculable in the $SU(2)_L \otimes U(1)$ model have the right sign and magnitude to render $|V_{ud}|^2 + |V_{us}|^2 < 1$ and restore unitarity.[1] This quantum-loop triumph for the standard model is, in a sense, an electroweak counterpart to the many $O(\alpha)$ successes of quantum electrodynamics ($g-2$, hyperfine splittings, etc.).

In this Letter we wish to refine still further the calculation of radiative corrections to β decay. Our goal is to push this $O(\alpha)$ test of the standard model to the level of a few tenths of a percent. Then, scrutinizing the three-generation unitarity constraint in Eq. (3), we will be able to place restrictions on new physics such as fourth-generation mixing or glean a signal of its presence. To that end, we will employ a recent detailed analysis of K_{e3} and hyperon decays by Leutwyler and Roos[3] that found

$$|V_{us}| = 0.221 \pm 0.002 \tag{4}$$

and combined measurements of the b-quark lifetime and bounds on $\Gamma(b \to u)/\Gamma(b \to c)$ which imply[3,4]

$$|V_{ub}| < 0.008. \tag{5}$$

We begin by summarizing the results of Ref. 1. There it was shown that after normalization of β-decay amplitudes in terms of $G_\mu = 1.166\,35 \times 10^{-5}\ \mathrm{GeV}^{-2}$, the muon decay constant, $O(\alpha)$ radiative corrections to superallowed $0^+ \to 0^+$ Fermi transition rates gave rise to an overall factor[1,5]

$$\left[1 + \frac{\alpha}{2\pi}\bar{g}(E_m)\right]\left\{1 + \frac{\alpha}{2\pi}\left[4\ln\left(\frac{m_Z}{m_p}\right) + \ln\left(\frac{m_p}{m_A}\right) + 2C + \mathscr{A}_g\right]\right\}, \tag{6}$$

which reduces $\mathscr{F}t$ values and $|V_{ud}|^2$ derived from them.[6] The first factor in Eq. (6), sometimes called the outer correction,[5] represents a spectrum-averaged effect that depends on the end-point β energy E_m. The remaining (inner) corrections are dominated by the large short-distance term $4\ln(m_Z/m_p)$ ($m_Z \simeq 93.2$ GeV, $m_p \simeq 0.938$ GeV). Also present is an axial-vector-induced structure-dependent contribution $\ln(m_p/m_A) + 2C$, where m_A is a low-energy cutoff applied to the short-distance part of the γW box diagram and $2C$ represents the remaining long-distance (low-frequency) correction. This term depends on the details of strong-interaction structure at low energy and is therefore the main source of uncertainty in the radiative corrections. Finally, $\mathcal{A}_g$ denotes small perturbative $O(\alpha_s)$ QCD corrections that can be rather reliably calculated.[1]

In previous extractions of $|V_{ud}|$ from measured $\mathcal{F}t$ values, the inner correction factor in Eq. (6) was generally taken to be 1.0210. From that value, a recent up-to-date analysis by Towner and Hardy which obtained a recommended $\mathcal{F}t = 3080.1 \pm 2.4$ sec leads to[7]

$$|V_{ud}| = 0.9742 \pm 0.0004, \tag{7}$$

where ± 0.0004 represents the error arising only from experimental and nuclear theory uncertainties. (Nuclear effects stem from isospin-breaking Coulomb corrections to transition matrix elements.[8])

To refine further the Towner and Hardy result and estimate the theoretical uncertainty in $|V_{ud}|$ due to radiative corrections, we have carried out the following reanalysis of Eq. (6).

(1) The small perturbative QCD correction $\mathcal{A}_g$, which was estimated to be -0.37 in Ref. 1, has been reduced slightly in magnitude to

$$\mathcal{A}_g = -0.34. \tag{8}$$

This change is due to a shift in $\sin^2\theta_W$ and $\alpha_s(1\text{ GeV})$ from 0.35 and 0.5 (used in Ref. 1) to more contemporary values of 0.22 and 0.3, respectively.[9]

(2) The structure-dependent term $\ln(m_p/m_A) + 2C$ arising from the axial-vector current was estimated by a procedure similar to the approach used by Marciano and Sirlin.[9] For the low-energy cutoff m_A, we allowed for a range 400 MeV $\leq m_A \leq$ 1600 MeV, while the low-energy part $2C$ was approximated by the Born contribution in two models of nuclear decay. In the first, following Dicus and Norton,[10] we considered the $0^+ \to 0^+$ decay of the nucleus as a whole in which case $C = 0$. In the second, we employed an independent-particle model of the process, in which case the Born contribution was analyzed by a new calculation involving nucleon electromagnetic and axial-vector dipole form factors.[11] We found $C = 3g_A(0.266)(\mu_p + \mu_n) = 0.885$ where $\mu_p + \mu_n = 0.88$ is the nucleon isoscalar magnetic moment. Allowing for the above ranges, and using a symmetric error, we estimate

$$\frac{\alpha}{2\pi}\left[\ln\frac{m_p}{m_A} + 2C\right] = 0.0012 \pm 0.0018. \tag{9}$$

(3) Since the $(2\alpha/\pi)\ln(m_Z/m_p)$ contribution is the largest $O(\alpha)$ correction in Eq. (6), we have approximated the effect of higher orders by summing all leading-logarithmic corrections of $O(\alpha^n \ln^n m_Z)$, $n = 1, 2, \ldots$, via a renormalization-group analysis. Such a summation replaces Eq. (6) with[12]

$$\left\{1 + \frac{\alpha}{2\pi}\left[\ln\left(\frac{m_p}{m_A}\right) + 2C\right] + \frac{\alpha(m_p)}{2\pi}[\bar{g}(E_m) + A_g]\right\} S(m_p, m_Z), \tag{10}$$

where

$$S(m_p,m_Z) = \left[\frac{\alpha(m_c)}{\alpha(m_p)}\right]^{3/4}\left[\frac{\alpha(m_\tau)}{\alpha(m_c)}\right]^{9/16}\left[\frac{\alpha(m_b)}{\alpha(m_\tau)}\right]^{9/19}\left[\frac{\alpha(m_t)}{\alpha(m_b)}\right]^{9/20}\left[\frac{\alpha(m_W)}{\alpha(m_t)}\right]^{3/8}\left[\frac{\alpha(m_Z)}{\alpha(m_W)}\right]^{12/11} \tag{11}$$

is a QED short-distance enhancement factor and $\alpha(\mu)$ is a running QED coupling defined by $\overline{\text{MS}}$ (modified minimal subtraction) which satisfies[12]

$$\mu\frac{d}{d\mu}\alpha(\mu) = b_0\alpha^2(\mu) + \text{higher orders}, \tag{12a}$$

$$b_0 = \frac{2}{3\pi}\sum_f Q_f^2\theta(\mu - m_f) - \frac{7}{2\pi}\theta(\mu - m_W) \tag{12b}$$

(sum over all elementary fermions),

$$\alpha^{-1}(0) = 137.036 + \frac{1}{6\pi} = 137.089. \tag{12c}$$

The values of $\alpha(\mu)$ to be used in Eq. (11) are found by integration of Eq. (12). That procedure gives[12]

$$\begin{aligned}
&\alpha^{-1}(m_Z = 93.2\text{ GeV}) = 127.66,\\
&\alpha^{-1}(m_W = 82.2\text{ GeV}) = 127.73,\\
&\alpha^{-1}(m_t = 40\text{ GeV}) = 128.95,\\
&\alpha^{-1}(m_b = 4.5\text{ GeV}) = 132.04,\\
&\alpha^{-1}(m_\tau = 1.78\text{ GeV}) = 133.29,\\
&\alpha^{-1}(m_c = 1.25\text{ GeV}) = 133.69,\\
&\alpha^{-1}(m_p = 0.938\text{ GeV}) = 133.93,
\end{aligned} \tag{13}$$

which when inserted into Eq. (11) yield

$$S(m_p, m_Z) = 1.022\,56. \tag{14}$$

Including the above refinements, we find in the case of ^{14}O (for example) where $\bar{g}(E_m)=11.107$ that Eq. (10) gives 1.0369 ± 0.0019 to be compared with 1.034 17 used in the Towner and Hardy analysis.[7] (The relative increase is essentially the same for all eight $0^+\to0^+$ transitions analyzed in Ref. 7.) Rescaling their result for $|V_{ud}|$ by $(1.034\,17/1.0369)^{1/2}$ and taking the conservative approach[13] of doubling the quoted uncertainty in Eq. (7), we find

$$|V_{ud}|=0.9729\pm0.0008\pm0.0009. \qquad (15)$$

The central value of $|V_{ud}|$ has been reduced by 0.13% and an estimate of the theoretical uncertainty, ±0.0009, in the inner corrections has been determined. Using this value along with the constraints in Eqs. (4) and (5) and combining errors in quadrature, we find

$$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=0.9954\pm0.0025. \qquad (16)$$

The result in Eq. (16) is consistent with three-generation unitarity at the 2-standard-deviation level and must continue to be viewed as a quantum-loop triumph for the $SU(2)_L\otimes U(1)$ model. Nevertheless, it is interesting to speculate as to what effect new physics would have on our analysis. In particular, the existence of a fourth fermion generation could manifest itself as a deviation from three-generation unitarity. Employing Eq. (16), we find that at 90% C.L. the fourth-generation mixing parameter $|V_{ub'}|$ is not very stringently bounded:

$$|V_{ub'}|<0.088\ (90\%\ \text{C.L.}). \qquad (17)$$

In fact, the central value in Eq. (16) corresponds to $|V_{ub'}|=0.068$ [compare with Eq. (5)]; but the error would have to be significantly reduced before one could argue that evidence for fourth-generation mixing had been found. Regarding the likelihood of a fourth generation, we remind the reader that such a scenario is not in conflict with any experiment. In fact, some grand unified theories actually predict[14] that there should be at least four relatively light generations. It is also interesting to note that recent papers by Anselm *et al.* and He and Pakvasa[15] have demonstrated how relatively large fourth-generation mixing could resolve problems with accommodating the *CP*-nonconservation parameters ϵ and ϵ' in the three-generation theory. It may also lead to detectable $\sim1\%$ D^0-$\bar{D}^0$ oscillations[15] and an enhancement in $K^+\to\pi^+\nu\bar{\nu}$. Clearly, a fourth generation with $|V_{ub'}|\sim6\%$ would be an exciting development with many interesting experimental implications.

If a fourth generation is not responsible for the small deviation from unity in Eq. (16), what is? The simplest explanation is that $|V_{ud}|$, $|V_{us}|$, or $|V_{ub}|$ (or more than one) is actually larger than the range of values we have allowed. In the case of $|V_{ub}|$, there is a tendency for the experimental b lifetime to be getting shorter than the value 1×10^{-12} sec used in obtaining Eq. (5).[4] If, in addition, $\Gamma(b\to u)/\Gamma(b\to c)$ actually exceeds the experimental bound 0.04, then $|V_{ub}|$ could increase beyond the bound in Eq. (5). However, it seems unlikely that $|V_{ub}|$ can be as large as 0.068 which is needed to bring the central value in Eq. (16) up to 1. Nevertheless, τ_b, $\Gamma(b\to u)/\Gamma(b\to c)$, and their mutual dependence on $|V_{ub}|$ need continued experimental and theoretical scrutiny.

A second (perhaps more likely) possibility is that $|V_{us}|$ is actually as large as 0.231 rather than in the range of Eq. (4). Indeed, hyperon β decays do favor a higher value.[3,16] Unfortunately, SU(3)-breaking effects in hyperon β decays are potentially large and not well under control.[3] More theoretical work is called for.

Finally, there may be additional uncertainties in the nuclear $0^+\to0^+\ \mathscr{F}t$ values. We have allowed somewhat for such a possibility by doubling the error quoted by Towner and Hardy. However, it is our belief that the $O(Z\alpha^2)$ corrections ($Z=$ charge of daughter nucleus) and the nuclear Coulomb corrections still need to be critically reexamined.[13] On the side of experiment, it seems unlikely that the precise data[17] employed by Towner and Hardy will shift very much. It would, however, be useful if comparable measurements could be carried out for $0^+\to0^+$ superallowed transitions in ^{10}C which has a (small) $Z=5$ daughter and hence relatively small Z-dependent effects. Of course, all Z-dependent corrections as well as much of the theoretical uncertainty can be eliminated by a precise high-statistics experiment on pion β decay $\pi^+\to\pi^0+e^++\nu_e$.[1,11] (The small branching ratio, 1×10^{-8}, makes such a measurement difficult.) A recent Los Alamos experiment[18] found $|V_{ud}|=0.962\pm0.018$ from pion beta decay. Although that result is in good agreement with Eq. (15), the experimental error is too large for it to be meaningful at the level of our analysis. However, it should be clear from our discussion that a new higher-statistics experiment is imperative and should be undertaken.[19,20]

In summary, our results demonstrate the importance of radiative corrections and precise experimental measurements. Superallowed $0^+\to0^+$ nuclear beta-decay $\mathscr{F}t$ values remain a quantum-loop triumph for the $SU(2)_L\otimes U(1)$ theory even at the several tenths of a percent level we have analyzed and provide a stringent constraint on new-physics appendages or competitors to the standard model. Future experiments and continued theoretical scrutiny will determine whether the small deviation from unity in Eq. (15) is evidence for new physics, such as a fourth generation, or merely the effect of additional theoretical uncertainties. To help resolve this issue and provide as precise a determination of $|V_{ud}|$ as possible, a new high-statistics pion beta-decay experiment would be particularly useful and we strongly urge such an undertaking.[19,20] Finally, we note that it is extremely important to lower the errors on[4] $|V_{cd}|=0.24\pm0.03$ and $|V_{cs}|=0.85\pm0.15$ by precise measurements of charm production and decay. Used in the unitarity constraints $|V_{ud}|^2+|V_{cd}|^2\leq1$ and $|V_{us}|^2+|V_{cs}|^2\leq1$, they can provide useful checks on $|V_{ud}|$ and $|V_{us}|$. More importantly, on the assumption that a reliable determination of $|V_{cb}|$ from b decay is forthcoming, the three-generation unitarity constraint $|V_{cd}|^2+|V_{cs}|^2+|V_{cb}|^2=1$ may also be used as a probe of fourth-generation mixing. Unitarity of the Kobayashi-Maskawa matrix is a powerful constraint that we are only beginning to exploit.

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[7]I. S. Towner and J. C. Hardy, in *Proceedings of the Seventh International Conference on Atomic Masses and Fundamental Constants*, edited by O. Klepper (Gesellschaft für Schwerionenforschung, Darmstadt, 1984), p. 564.

[8]Earlier estimates of $|V_{ud}|$, e.g., R. E. Shrock and L.-L. Wang, Phys. Rev. Lett. **41**, 1692 (1978); D. H. Wilkinson, Ref. 6; M. A. B. Bég and A. Sirlin, Phys. Rep. **88**, 1 (1982); E. Paschos and U. Turke, Phys. Lett. **116B**, 360 (1982), found a somewhat lower value than Eq. (7) mainly because they employed smaller nuclear Coulombic corrections. For a discussion see Ref. 6 and I. S. Towner and J. C. Hardy, Phys. Lett. **73B**, 20 (1978).

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[11]A free-nucleon calculation by S. M. Berman and A. Sirlin, Ann. Phys. (N.Y.) **20**, 20 (1962), gave $C=\frac{9}{8}$. In the case of pion beta decay, $C=0$ in the Born approximation, but axial-vector-meson intermediate states are expected to contribute at the level of $C\sim0.1$; see Ref. 1. Improvement of this estimate may, in the future, become an important challenge to lattice gauge theorists.

[12]$S(m_p,m_Z)$ is obtained by the solving of the renormalization-group equation

$$\left[\mu\frac{\partial}{\partial\mu}+b_0\alpha^2\frac{\partial}{\partial\alpha}-\frac{2}{\pi}\alpha\right]S(m_p,\mu)=0$$

and using $S(m_p,m_p)=1$. See W. Marciano and A. Sirlin, Phys. Rev. Lett. **46**, 163 (1981).

[13]We have doubled the error quoted in Eq. (7) because a recent analysis of ^{34}Cl by W. E. Ormand and B. A. Brown, Nucl. Phys. **A440**, 274 (1985), suggests that the nuclear Coulombic corrections are actually smaller than the values employed in Ref. 7. In addition, we feel that certain aspects of the $O(Z\alpha^2)$ corrections [see W. Jaus and G. Rasche, Nucl. Phys. **A143**, 202 (1970)] need clarification.

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[15]A. A. Anselm *et al.*, Phys. Lett. **156B**, 102 (1985); X.-G. He and S. Pakvasa, Phys. Lett. **156B**, 236 (1985).

[16]M. Bourquin *et al.* (WA2 Collaboration), Z. Phys. C **12**, 307 (1982), obtained $|V_{us}|=0.231\pm0.003$ from hyperon beta decay. More recently, A. Bohm and M. Kmiecik, Phys. Rev. D **31**, 3005 (1985), included new Fermilab hyperon data and found $|V_{us}|=0.225\pm0.002$. Unfortunately, SU(3)-breaking effects are potentially large and not reliably calculable (see Ref. 3); so it is difficult to assess their effect on these results.

[17]V. T. Koslowsky *et al.*, in *Proceedings of the Seventh International Conference on Atomic Masses and Fundamental Constants*, edited by O. Klepper (Gesellschaft für Schwerionenforschung, Darmstadt, 1984), p. 572.

[18]W. K. McFarlane *et al.*, Phys. Rev. Lett. **51**, 249 (1983), and Phys. Rev. D **32**, 547 (1985).

[19]The uncertainty in $m_{\pi^+}-m_{\pi^0}$ presently gives rise to ±0.0020 uncertainty in $|V_{ud}|$. Presumably, that could be lowered if a high-precision pion beta-decay experiment were undertaken.

[20]A precise measurement of $K_L^0\rightarrow K^\pm e^\mp\overset{(-)}{\nu}$ could also be used to determine $|V_{ud}|$ with little theoretical uncertainty. The expected branching ratio $\simeq0.5\times10^{-8}$ makes such an experiment feasible at future high-flux kaon factories; but the clean identification of such decays would be difficult. See V. Highland, in "Physics with LAMPF II," Los Alamos National Laboratory Report No. LA-9798-P, 1983 (unpublished), p. 302. We thank M. Goldhaber and L. Littenberg for discussions on this decay mode.